Prime Number Generating Function

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For any positive z, let

$$f(z) = \frac{\pi z}{\sin(\pi z)} \cdot \prod_{k=2}^{\infty} \frac{\sin(\pi z/k)}{\pi z/k}$$

We have

$$f(z) = \frac{1}{1-z^2} \cdot \prod_{k=2}^{\infty} \Big\{ \frac{\sin(\pi z/k)}{\pi z/k} \cdot \frac{1}{1-z^2/k^2} \Big\}$$

Thus

$$\begin{aligned} f(z) &= 0 \text{ if } z \text{ is composite} \\ f(z) &\neq 0 \text{ if } z \text{ is prime} \\ f(z+1) &= 0 \text{ if } z \text{ is prime} \end{aligned}$$

Now let

$$g(z) = \frac{\big\{f(z+1)\big\}^2}{\lambda\big\{f(z)\big\}^{2\rho} + \big\{f(z+1)\big\}^2}$$

where $\lambda > 0, \rho > 0$ and $\rho \geq \sqrt{2z}$. When both z and $z+1$ are composite, both the numerator and denominator vanish. In this case, the above expression for $g(z)$ should be interpreted as a limit. The exponent ρ guarantees that this limit is equal to 1. Note that $g(z)$ is a continuous function for $z > 0$. Its first derivative exists and is finite except when z is an integer. Finally, we have:

$$\begin{aligned} g(z) &= 0 \text{ if } z \text{ is prime} \\ g(z) &= 1 \text{ if } z \text{ is composite} \\ g(z) &\in \,]0,1[\text{ otherwise} \end{aligned}$$

Thus finding all the prime numbers consists of solving $g(z) = 0$.